

## TPS Decision Problem

## - Decide to implement TPS Option B in-house

- Two primary candidates in developing OS
- Option B-Conservative (BC)
- Sure to work
- Cost $=\$ 260 \mathrm{~K}$
- Only achieve a peak performance of $160 \mathrm{tr} / \mathrm{sec} \mathrm{w} / 8$ processor configuration
- Option B-Bold (BB)
- Use of the recently developed hypermonitor concept
- Cost $=\$ 260 \mathrm{~K}$
- Achieve a peak performance of $190 \mathrm{tr} / \mathrm{sec} \mathrm{w} / 8$ processor configuration
- If not successful, reprogram w/ Option BC, achieving 160 tr/sec and a added cost $\$ 60 \mathrm{~K}$



## Payoff Matrix: Option BC \& BB

| Alternative | State of Nature |  |
| :---: | :---: | :---: |
|  | Favorable | Unfavorable |
| BB (Bold) | 150 | -30 |
| BC (Conservative) | 30 | 30 |

## Decision Rules for Complete Uncertainty

## Maximin Rule

- The most pessimistic decision rule
- Determine the minimum payoff for each alternative. Choose the alternative which maximizes the minimum payoff.

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## Laplace (Equal-Probability) Rule

- Recognizes the relative magnitude of the payoff
- Assume all of the states of nature are equally likely. Determine the expected value for each alternative, and choose the alternative with the maximum expected value

| Alternative State of Nature  Expected Value <br> (EV) <br>  Favorable   <br>  Unfavorable   |  |  |  |
| :--- | :---: | :---: | :---: |
|  | 150 | 0.5 | -30 |
|  | 30 | 30 | 60 |

## Problems with Laplace Rule

|  | Favorable | $U_{1}$ | $U_{2}$ | EV |
| :---: | :---: | :---: | :---: | :---: |
| BB | 150 | -30 | -30 | 30 |
| BC | 30 | 30 | 30 | 30 |

$U_{1}$ : Performance Failure of hypermonitor and $U_{2}$ : Reliability Failure

- subject to pitfalls, such as duplication of the states of nature
- Nothing has changed in the real world situation, But, our labeling of the states of nature causes a significant change in the expected value and the recommended decision
- because of the equal-probability assumption


## Breakeven Analysis

- Treat uncertainty as a parameter: $\mathrm{P}=\mathrm{Prob}$ (favorable)
- Compute expected values as $f(P)$
- EV (BB) $=P(250)+(1-P)(-50)=-50+300 P$
$-E V(B C)=P(50)+(1-P)(50)=50$
- Determine breakeven point $P$ such that
- $\mathrm{EV}(\mathrm{BB})=\mathrm{EV}(\mathrm{BC}):-\mathbf{5 0}+\mathbf{3 0 0 P}=\mathbf{5 0}, \mathrm{P}=0.333$



## Utility Function

- Suppose you are a manager presented with the following two choices:
- Option 1: guaranteed payoff of $\$ 60 \mathrm{~K}$
- Option 2: $50 \%$ chance of a payoff of $\$ 150 \mathrm{~K}$, and a $50 \%$ chance of a loss of \$30K
- Which one would you prefer?
- The same expected values



## Software Engineering Implications

- Managers prefer loss-aversion
- EV-approach unrealistic with losses
- Managers’ U.F.'S linear for positive payoffs
- Can use EV-approach then
- People's U.F.'S aren’t
- Identical
- Easy to predict
- Constant


## Statistical Decision Theory: The Value of Information

## TPS: Decision under Uncertainty

| Alternative | State of Nature |  |
| :--- | :---: | :---: |
|  | Favorable | Unfavorable |
| BB (Bold) | 150 | -30 |
| BC (Conservative) | 30 | 30 |

- Available decision rules are inadequate
- Absence of any information about the probable occurrence of the states of nature
- Need better information
- Information $\rightarrow$ Considerable Economic Value in decision problems under uncertainty
- Build a Prototype for \$10K
- If prototype succeeds, choose BB

Payoff: \$150K - 10K = \$140K

- If prototype fails, choose BC

Payoff: \$30K - 10K = \$20K

- If equally likely,

$$
\text { Ev = } 0.5(\$ 140 K)+0.5(\$ 20 K)=\$ 80 K
$$

- Could invest up to $\$ 30 \mathrm{~K}$ and do better than before
- thus, EVPI = \$30K


## Working with Imperfect Information

- Can not obtain perfect Information
- By a Prototype or other investigations
- Two Sources of Imperfection expressed as probabilities
$P(I B \mid S F) \neq 0.0$
Investigation (Prototype) would lead us to choose bold alternative in a state of nature in which the Bold option will fail
$P(I B \mid S S) \neq 1.0$
Investigation (Prototype) would lead us to choose bold alternative in a state of nature in which the Bold option will succeed


## Example

- Suppose we assess the prototype's imperfections as $\mathbf{P}(\mathrm{IB} \mid \mathbf{S F})=\mathbf{0 . 2 0}$,

$$
\mathrm{P}(\mathrm{IB} \mid \mathrm{SS})=0.90
$$

- Suppose the states of nature are equally likely

$$
\mathrm{P}(\mathrm{SF})=0.50 \quad \mathrm{P}(\mathrm{SS})=0.50
$$

- Compute the expected value of using the prototype EV(IB,IC) = P(IB) (Payoff if use Bold)
+ P(IC) (Payoff if use Conservative)

$$
=\mathbf{P}(\mathrm{IB})[\mathrm{P}(\mathrm{SS} \mid \mathrm{IB})(\$ 150 \mathrm{~K})+\mathrm{P}(\mathrm{SF} \mid \mathrm{IB})(-\$ 30 \mathrm{~K})]+\mathrm{P}(\mathrm{IC})(\$ 50 \mathrm{~K})
$$

But these are the probabilities we don't know

## How to get the probabilities

$$
\begin{aligned}
& P(I B)=P(I B \mid S S) P(S S)+P(I B \mid S F) P(S F) \\
& P(I C)=1-P(I B)
\end{aligned}
$$

$$
P(S S \mid I B)=\frac{P(I B \mid S S) P(S S)}{P(I B)}
$$


$P(S F \mid I B)=1-P(S S \mid I B)$
$P\left(S S|\mid B)=\frac{\text { Prob(we will choose Bold in a state of nature where it will succeed) }}{\text { Prob(we will choose Bold) }}\right.$

## Expected Value for TPS

$$
\begin{aligned}
& P(I B)=(0.50) P(0.90)+(0.50) P(0.20)=0.55 \\
& P(I C)=1-0.55=0.45 \\
& P(S S \mid I B)=\frac{(0.50)(0.9)}{0.55}=0.82 \\
& P(S F \mid I B)=1-(0.82)=0.18 \\
& E V(I B, I C)=(0.55)[(0.82)(\$ 105 K)+(0.18)(\$-30 K)]+(0.45)(\$ 30 K) \\
& =(0.55)(\$ 117.6 K)+\$ 13.5 K=\$ 78.2 K
\end{aligned}
$$

## Net Expected Value Maximization

| Cost of Prototype, <br> $\$ K$ | Estimated |  | Expected <br> Value, $\$ \mathrm{~K}$ | Expected Value <br> of Information | Net <br> Expected <br> Value, $\$ \mathrm{~K}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P}(\mathrm{PS} \mid \mathrm{SF})$ | $\mathrm{P}(\mathrm{PS} \mid \mathrm{SS})$ |  | 60 | 0 |
| 0 |  |  |  |  |
| 0 |  |  | 69.3 | 9.3 | 4.3 |
| 5 | 0.30 | 0.80 | 0.90 | 78.2 | 18.2 |
| 10 | 0.20 | 0.2 | 8.2 |  |  |
| 20 | 0.10 | 0.95 | 86.8 | 26.8 | 6.8 |
| 30 | 0.00 | 1.00 | 90 | 30.0 | 0 |

## Value of Information in SE

## - Value of Information Procedure

- Can help to resolve a number of key software engineering decisions
"How much should we invest in further information gathering and analysis investigations before committing ourselves to a course of action"
- Four major issues of this nature in SE

How much should we invest in:

- Feasibility studies
- Alternative vendor $\mathbf{h} / \mathrm{w}-\mathrm{s} / \mathrm{w}$ product analysis
- Risk analysis
- V \& V


## Conditions for Successful Prototyping

1. There exist alternatives whose payoffs vary greatly depending on some states of nature.
2. The critical states of nature have an appreciable probability of occurring.
3. The prototype has a high probability of accurately identifying the critical states of nature.
4. The required cost and schedule of the prototype does not overly curtail its net value.
5. There exist significant side benefits derived from building the prototype.

## Pitfalls Avoided by Using VOI

1. Always build a prototype or simulation

- May not satisfy conditions 3,4

2. Always build the software twice

- May not satisfy conditions $\mathbf{1 , 2}$

3. Build the software purely top-down

- May not satisfy conditions 1,2

4. Prove every piece of code correct

- May not satisfy conditions $1,2,4$

5. Nominal-case testing is sufficient

- May need off-nominal testing to satisfy conditions 1,2,3
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